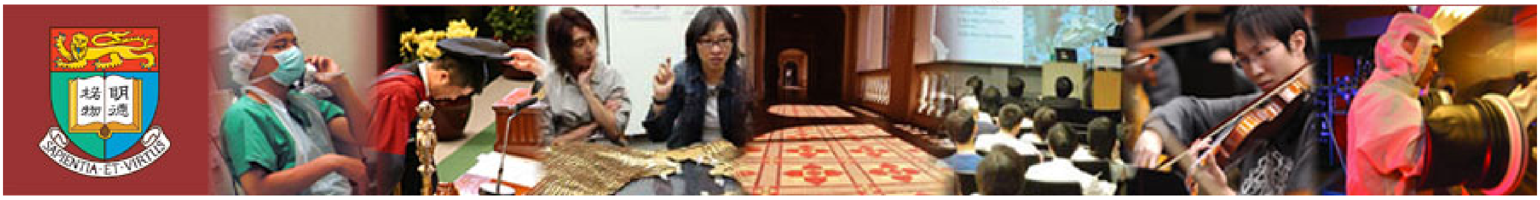

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Accelerated A-EFIE with Perturbation Method Using Fast Fourier Transform

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***Abstract*—In this paper, we apply the fast Fourier transform on the perturbation-based augmented electric field integral equation (A-EFIE). The main idea lies on the Lagrangian interpolation of a series $R^n, n = -1, 0, 1, 2, ...$, obtained by expanding Green's function using Taylor series. By utilizing the Toeplitz property of the R^n on the uniform cartesian grids, the multiplication of expanded kernels in the vector and scalar potentials can be accelerated effectively. The oscillation of these expanded kernels has less variation compared to the original kernel with e^{ikR}/R. In addition, we do not need to do any near field amendment when $n \geq 0$. Numerical example validates the feasibility and validity of this method.**

I. Introduction

It is well-known that there is a low-frequency breakdown problem in electric field integral equation (EFIE) when Rao-Wilton-Glisson (RWG) basis functions are employed in moments method. In the past years, different solutions to this low-frequency problem have been proposed, such as Helmholtz decomposition of basis functions [1], [2] and Calderón preconditioner [3]. Among them, the augmented electric field integral equation (A-EFIE) becomes one of the most effective methods to remedy the low-frequency breakdown [4], [5], which is the only method independent of the basis functions. Later on, a perturbation method was introduced to enhance the accuracy of A-EFIE [6]. However, the perturbation method generates more matrix vector products at different frequency orders, which have to be accelerated before solving the real-world problems.

As the number of unknowns increases, a fast algorithm has to be incorporated into the iterative solver. The most popular multilevel implementation of fast multipole algorithm (MLFMA) [7], the mixed-form FMA [8], LF-FMA algorithm [9] and accelerated cartesian expansion (ACE) with FMM [10] have been used to accelerate the original kernel of A-EFIE. However, the computational cost becomes extremely high when the kernel is expanded into the form of $R^n, n = -1, 0, 1, 2, ...$

Unlike the other fast algorithms, fast Fourier transform (FFT) based method is another class of fast integral equation methods [11], [12]. Because of its simplicity and kernel independent property, it is suitable to accelerate the expanded kernel of $R^n, n = -1, 0, 1, 2, ...$ (Green's function's Taylor expanding series) instead of e^{ikR}/R. In this work, we expand these series using Lagrangian interpolation. The oscillation of the kernels in this method has less vibration compared to the original kernel with e^{ikR}/R. It is important to notice that near field amendment is not needed when $n \geq 0$.

II. Backgroud

For a perfect electric conductor (PEC) surface S', the perturbation based A-EFIE in its mixed potential form can be written as

$$\begin{bmatrix} \bar{\mathbf{V}} & \bar{\mathbf{D}}^T \cdot \bar{\mathbf{P}} \cdot \bar{\mathbf{B}} \\ \bar{\mathbf{F}} \cdot \bar{\mathbf{D}} & k_0^2 \bar{\mathbf{I}}_r \end{bmatrix} \cdot \begin{bmatrix} ik_0 \mathbf{j} \\ c_0 \rho_r \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix} \tag{1}$$

Under the condition $|ik_0 R| \ll 1$, the Green's function can be expanded by Taylor series (Maclaurin series),

$$g(\mathbf{r}, \mathbf{r}') \approx \frac{1}{4\pi R}(1 + ik_0 R + \frac{1}{2!}(ik_0 R)^2 + \frac{1}{3!}(ik_0 R)^3 + \cdots \frac{1}{n!}(ik_0 R)^n + \cdots) \tag{2}$$

The perturbation method was proposed to remedy the low frequency inaccuracy for scattering and capacitive problems [6]. Letting $\delta = ik_0$, the vector (scalar) potential sub-matrix can be expanded as

$$\bar{\mathbf{V}} = \bar{\mathbf{V}}^{(0)} + \delta \bar{\mathbf{V}}^{(1)} + \delta^2 \bar{\mathbf{V}}^{(2)} + \delta^3 \bar{\mathbf{V}}^{(3)} + \cdots \delta^n \bar{\mathbf{V}}^{(n)} + \cdots \tag{3}$$

$$\bar{\mathbf{P}} = \bar{\mathbf{P}}^{(0)} + \delta \bar{\mathbf{P}}^{(1)} + \delta^2 \bar{\mathbf{P}}^{(2)} + \delta^3 \bar{\mathbf{P}}^{(3)} + \cdots \delta^n \bar{\mathbf{P}}^{(n)} + \cdots \tag{4}$$

where

$$\left[\bar{\mathbf{V}}^{(N)}\right]_{mn} = \mu_r \int_{S_m} \mathbf{\Lambda}_m(\mathbf{r}) \cdot \int_{S_n} g^{[N]}(\mathbf{r}, \mathbf{r}') \mathbf{\Lambda}_n(\mathbf{r}') dS' dS \tag{5}$$

$$\left[\bar{P}^{(N)}\right]_{mn} = \frac{1}{\varepsilon_r} \int_{S_m} h_m(\mathbf{r}) \cdot \int_{S_n} g^{[N]}(\mathbf{r}, \mathbf{r}') h_n(\mathbf{r}') dS' dS \tag{6}$$

For the perturbation method, we can expand Green's function at different orders into Cartesian Lagrange polynomial as,

$$g^{[N]}(\mathbf{r}, \mathbf{r}') = \sum_{n=0}^{N_g - 1} \sum_{n=0}^{N_g - 1} \beta_n^p(\mathbf{r}) g_{n,n'}^{[N]} \beta_{n'}^p(\mathbf{r}') \tag{7}$$

where p is the order of Lagrange polynomial interpolation, N_g is the number of grid points [12], and

$$g^{[N]}(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \frac{R^{n-1}}{n!} \tag{8}$$

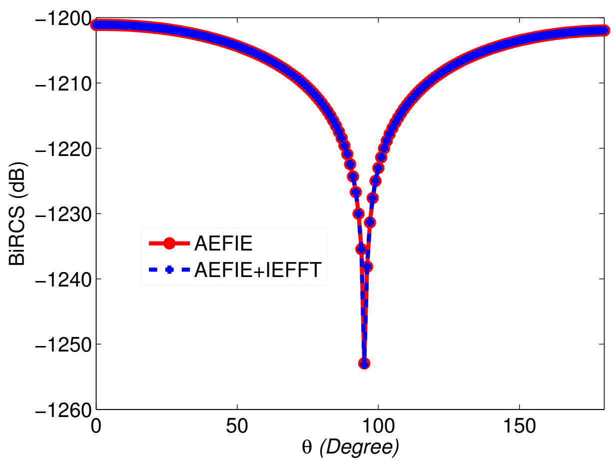

Fig. 1. Comparison of the bi-static RCS of the VFY218 for the vertical polarization calculated by A-EFIE and A-EFIE+FFT when the frequency is 10^{-25} Hz

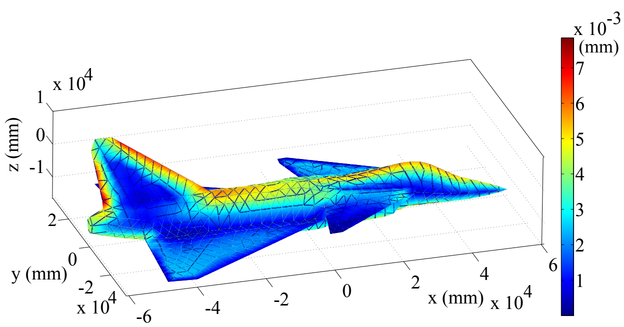

Fig. 2. The first order of current distribution for VFY218 calculated by A-EFIE+FFT

. It is important to find that we only need to do near-interaction amendment when $N = 0$, because of the singularity when the grids are overlapped. For the other cases, the matrix vector in (1) can be expressed as,

$$\bar{V}^{(n)} \cdot j^{(n)} = \mu_r \Pi_v \cdot F^{-1}\{F(g^{[n]}) \cdot F({\Pi_v}^T \cdot j^{(n)})\}(n \geq 1) \quad (9)$$

where Π_v and Π_s are the mapping matrices.

III. Numerical Results

To verify our proposal, an numerical example is tested in this section. Considering a PEC airplane VFY218, it is discretized into 22219 triangle patches and 33327 equivalent edges, the operation frequency is set at 1.0×10^{-25} Hz. Fig. 1 shows comparison of the RCS calculated by A-EFIE and A-EFIE+FFT, where excellent agreement is achieved with each other. The first order current distribution of this airplane is shown in Fig. 1. We can clearly find a current loop along the surface, which is well align with the explanation in [6]

The computational time for the matrix vector product and the total memory usage are tabulated in Table I.

TABLE I
Computational Cost of proposed method with GMRES-30 and Tolerance 10^{-7}.

methods	solving matrix equation times(s)	total memory (MB)	iteration numbers
AEFIE	—-	29618.90	262
AEFIE+FFT	3511.61	241.27	262

IV. Conclusion

The FFT technique has been adopted to accelerate the expanded integral equation kernels in the A-EFIE with perturbation method. It makes the perturbation method possible to efficiently solve the complicated real world problems with over one million unknowns without using a parallel computer. The effectiveness and accuracy of the proposed method has been verified by the given numerical example.

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